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Reentry Time Prediction Using Atmospheric Density Corrections

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Errors in the upper atmosphere density models have a significant influence on the accuracy of orbit prediction and, specifically, on the accuracy of the prediction of the reentry time of space objects. The determination of current time corrections to the atmosphere density and their use in orbit prediction are proposed as a method for increasing the accuracy of reentry time prediction. The potential effect of increasing the accuracy of space object reentry time prediction, associated with accounting for the corrections to the Naval Research Laboratory MSIS-00 atmosphere density model, is estimated for space objects having both spherical and nonspherical shapes. The results show that the reentry time predictions obtained using this approach are significantly improved. The improvement is better for spherical objects than arbitrary shaped objects due to the time varying nature of the ballistic coefficient of arbitrary shaped objects. The use of the atmospheric density corrections provides insight into the time variations of individual space object aerodynamic characteristics and allows their use in predicting the reentry time.

Nomenclature

b_1, b_2	=	coefficients used for the density correction
$F_{10.7}$	=	average daily solar flux at 10.7 cm wavelength
h	=	space object altitude
K_p	=	geomagnetic index
Kb	=	ballistic coefficient, m^2/kg
Kbf	=	ballistic coefficient after density corrections
t_{calc}	=	calculated (predicted) reentry time
t_{real}	=	actual reentry time
t_0	=	prediction epoch
$\delta\rho$	=	atmospheric density correction
ε	=	reentry parameter
ρ	=	atmospheric density
σ	=	standard deviation of the density error, %

I. Introduction

ATMOSPHERIC density mismodeling was, and remains, the dominant error source in the orbit determination and prediction of low-Earth-orbit (LEO) satellites. To improve the accuracy of motion prediction for these satellites, it has been proposed to track the actual density of the upper atmosphere using the available drag data on the cataloged LEO satellites. The total number of such drag-perturbed space objects (SOs) reaches several hundred at any given time. The element sets for these SOs are updated as an ordinary routine operation by the space surveillance systems. We use these element sets as the observation data for estimating the corrections between the actual atmosphere density and a chosen atmosphere density model. Recently we obtained the density corrections [1] for the Naval Research Laboratory MSIS-00 (NRLMSIS-00) atmosphere model [2] using the two-line element sets (TLEs) as observations. Time series for the density corrections were generated on a one-day grid over a four-year interval from 1 December 1999 to 30 November 2003. Figure 1 illustrates the time histories of the estimated correction parameters b_1 and b_2 for the NRLMSIS-00 density model. Using these data, one can independently estimate the corrections to the NRLMSIS-00 model density ρ and take them into account in orbit calculations using

$$\frac{\delta\rho}{\rho}(h,t) = b_1(t) + b_2(t)(h-400)/200 \tag{1}$$

where h is the altitude in kilometers.

The effectiveness of this process was evaluated by comparing the orbit determination and prediction results obtained without, and then with, the constructed density corrections. The application of the density corrections to the NRLMSIS-00 model reduced the scattering of the ballistic coefficient estimates by a factor of 2 for eccentric orbits and up to a factor of 5.6 for near-circular orbits. The reduced scattering in the ballistic coefficient values indicates that the various complexities in the physics of the atmosphere are more consistently modeled with the density corrections. For the Russian GOST atmosphere model [3], similar results were obtained previously [4]. However, it is necessary to note that these corrections mainly account for unmodeled density variations at altitudes from 300 to 600 km. These estimates of the density correction effects cannot be extended to decaying SOs for the following reasons:

1) The deficiency of the observation data used for the construction of the density corrections at low altitudes. This fact is illustrated by Fig. 2, in which the time–altitude distribution of the drag observation data is given for all of the 16 uncontrolled ballistic reentering space objects used for the construction of the density corrections for the NRLMSIS-00 model. It is seen that at altitudes lower than 280 km no drag measurements were obtained for some time intervals.

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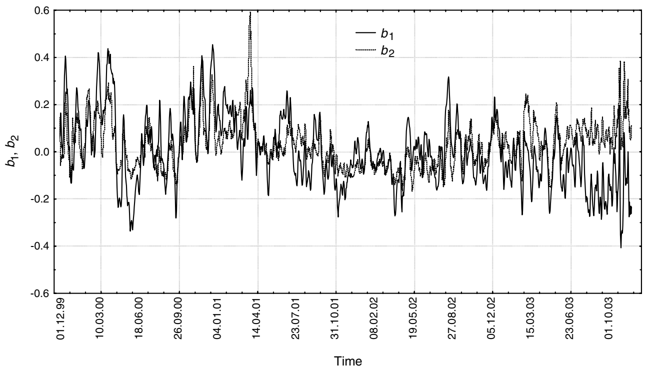

Fig. 1 Estimated b_1 and b_2 density correction parameters for the NRLMSIS-00 model.

2) The observed tendency of the relative errors in the density models to decrease with lower altitude. We obtained the altitude regressions for the rms of the model relative errors using the constructed corrections at altitudes from 200 to 600 km. For the NRLMSIS-00 model, this regression relationship is:

$$\sigma(h) = 1.04 + 0.0373h \tag{2}$$

where $\sigma(h)$ is expressed in percent and the altitude h is expressed in kilometers.

3) The increase in the influence of the SO's attitude on the accuracy of motion prediction for reentering objects. For the majority of SOs at high altitudes this influence is small compared to the unmodeled atmospheric density variations. However, if the SO orientation is not stabilized along the direction of incident airflow, the amplitude of the ballistic coefficient variations due to the SO rotation around the center of mass can increase with orbit decay, as compared to the unmodeled atmospheric density variations.

4) Time dependence of density variations. It is not improbable, on the time interval corresponding to the final stage of SO orbital flight, that the density corrections have near-zero values. In this case it is senseless to expect any positive effect by accounting for the density corrections.

5) Nonlinear effects. Unlike high altitudes, the atmospheric drag effect at low altitudes becomes so considerable that the character of the atmospheric density variations and the SO's orbital elements becomes essentially nonlinear. At higher altitudes, the atmospheric density values in the orbit determination fit and predict intervals do not differ as significantly as at lower altitudes. The linear model for the density corrections that we have used [Eq. (1)] may be unsuitable for the very low altitudes associated with reentry.

This paper studies the influence of unmodeled density variations for the NRLMSIS-00 model and variations in specific SOs' aerodynamic characteristics on the accuracy of reentry predictions. Section II describes the approach used. The results from applying the method to spherical and nonspherical objects are provided in Secs. III and IV. A discussion and results of the time varying nature of the ballistic coefficients for nonspherical objects is presented in Sec. V.

II. Approach Description

The method for investigating the reentering objects is combined with the method for monitoring the atmospheric density variations [4]. The flowchart describing this approach is given in Fig. 3.

Now consider in detail the elements of the technical approach:

1) To estimate the influence of unmodeled density variations on the errors in the reentry time prediction, it is necessary to have a sufficiently large set of statistical data obtained under the various conditions of the solution of the given task. Therefore, the acquisition of real orbital data in the TLE format for several tens of space objects that decayed from 2000–2003 was organized. The element sets for the chosen SOs were downloaded from the NASA Orbital Information Group (OIG)[¶] and the CelesTrak[**] Web sites. The TLE sets were transformed into osculating orbital elements, which were then considered as noisy "measurements" during the "smoothed" orbit and associated ballistic coefficient calculations.

2) The solar and geomagnetic activity indices were downloaded from the National Oceanic and Atmospheric Administration National Weather Service Space Environment Center (SEC) file-transfer protocol server[††].

3) For each measurement epoch, the smoothed orbit and the associated ballistic coefficient Kb were determined by a least-squares fit of the measurements created by the transformation of the TLEs. This least-squares fit process is called "secondary data processing." A set of measurements corresponding to the time interval before the epoch of the smoothed orbit was chosen for each fit. The fit interval used for the estimation of the smoothed elements and the associated ballistic coefficient depends on the satellite lifetime.

4) The complete version of the NRLMSIS-00 density model was used as a baseline model of the upper atmosphere density. The Everhart numerical method [5] was used for the propagation of the satellite motion and the reentry time estimation.

5) Similar to step 3, the smoothed orbit and associated ballistic coefficients were determined a second time for each epoch of the measurements by taking into account the density corrections. For altitudes lower than 180 km, instead of Eq. (1) the following equation was used for the calculation of the density corrections:

$$\delta\rho/\rho = [b_1(t) + b_2(t)(h - 400)/200] \exp[(h - 180)/30] \tag{3}$$

6) The influence of the unmodeled density variations was evaluated by comparing the reentry prediction results obtained without and with the estimated density corrections. To obtain the comparable error statistics across all satellites and for different prediction intervals, a normalized reentry error parameter, which is the ratio of the reentry prediction error to the residual lifetime value, is used

$$\varepsilon = \frac{\Delta t}{\text{lifetime}} = \frac{t_{\text{real}} - t_{\text{calc}}}{t_{\text{real}} - t_0} \tag{4}$$

where t_{real} and t_{calc} are the actual and predicted reentry times and t_0 is the epoch.

7) The a posteriori model of the calculations was used, that is, both on the measurements fitting interval and on the prediction interval. That is, the values of the solar activity and the geomagnetic index, as well as density corrections, were assumed to be known. In view of this circumstance, the obtained results can be considered as optimistic.

8) For the majority of the chosen 95 SOs, the reentry time calculation was started when the remaining lifetime became less than 10–18 days.

9) The true values of the reentry times for the chosen SOs were obtained from several Web sites: a) NASA OIG Web site: http://oig1.gsfc.nasa.gov, b) CelesTrak Web site: http://www.celestrak.com, c) http://www.aero.org/capabilities/cords/.html, d) http://wingar.demon.co.uk/satevo/index.htm, and e) http://www.satobs.org/seesat/index.html.

III. Spherical Space Objects

At low altitudes, prediction errors caused by the attitude motion of an SO may become comparable with, and even dominate, the effects from the unmodeled variations of the atmospheric density. To avoid mixing these two factors, we first consider the influence of the density corrections on the motion prediction characteristics of the

[¶]Data originally available online at http://oig1.gsfc.nasa.gov, which ceased operations on 31 March 2005; similar information is now provided at http://www.space-track.org, a U.S. Headquarters Air Force Space Command Web site.

[**]Data available online at http://www.celestrak.com.

[††]Data available online at http://dawn.sec.noaa.gov.

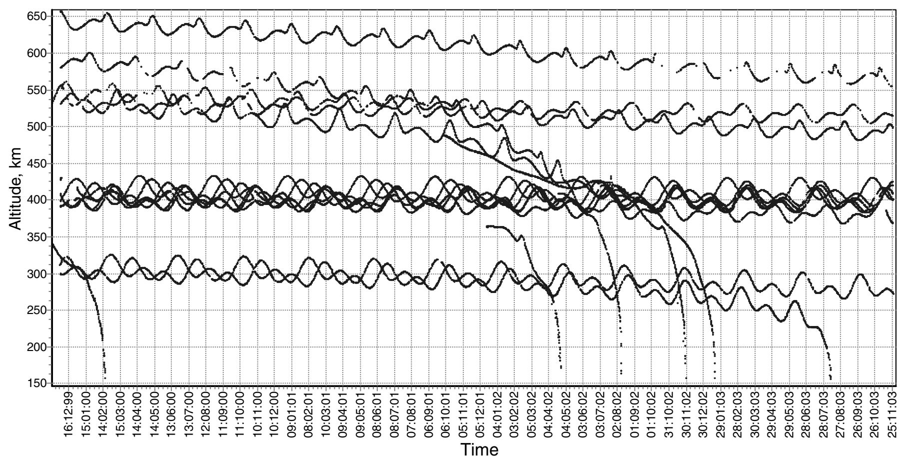

Fig. 2 Time-altitude distribution of observation data.

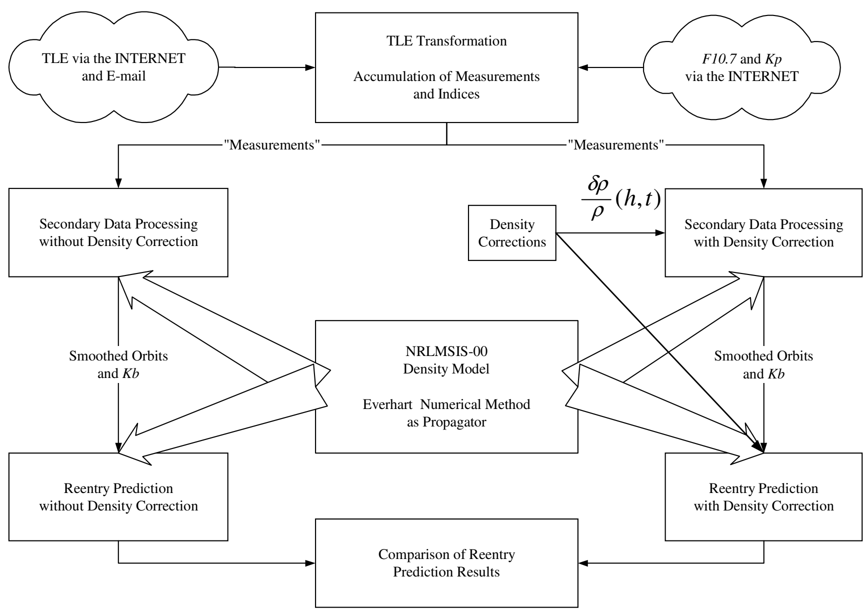

Fig. 3 Flowchart describing the approach.

spherical space objects. It is assumed that rotation around the center of mass does not change the ballistic coefficients of these SOs. Among the chosen SOs, only the satellites of the STARSHINE series were spherical. The characteristics of these satellites are presented in Table 1. The true decay times in Table 1 are the median of the reentry times obtained from the a, c, d, and e Web sites listed in the previous section.

Now consider in detail the results of the reentry time predictions for the STARSHINE 1 satellite. The STARSHINE 1 satellite decayed on 18 February 2000. Figure 4 shows the relative density corrections for the NRLMSIS-00 model from 1–19 February 2000. These corrections are plotted for the altitude range of 100–600 km, with a step of 100 km. Figure 5 shows the time history of the perigee altitude of the satellite, and Fig. 6 presents the estimates of the predicted reentry time for the final stage of the STARSHINE 1 lifetime. The reentry altitude was chosen to be 30 km. These estimates were obtained without and with density corrections for the NRLMSIS-00 model. In Fig. 6, the abscissa gives the epoch of the predicted orbit, and the ordinate gives the predicted values of the reentry time. The data presented in Figs. 4–6 indicate that at the beginning of February 2000 the model density values at the perigee altitude of the STARSHINE 1 satellite were 10% greater than the real values of density. The density corrections were negative over all altitudes at that time. As the STARSHINE 1 orbit decayed, the actual density at its flight altitude smoothly increased. Therefore, the reentry time predicted without density corrections gradually approached the reentry time predicted with density corrections (see Fig. 6).

Table 1 STARSHINE satellites of spherical shape

SO no.	Name	Inclination, deg	True decay time (coordinated universal time)
25769	STARSHINE 1	51.6	18 Feb. 2000, 15 h, 41 m
26929	STARSHINE 3	67.0	21 Jan. 2003, 05 h, 05 m
26996	STARSHINE 2	51.6	26 April 2002, 11 h, 11 m

Figure 7 presents the histogram of the relative error distribution [see Eq. (4)] for the STARSHINE 1 satellite. It can be seen from this plot that the application of the density corrections led to a decrease in the rms of the relative error from 8.1 to 1.4%.

Table 2 generalizes the data characterizing the influence of the density corrections to the NRLMSIS-00 model to include all the STARSHINE satellites. Therefore, the data of Table 2 indicate the following:

1) The application of density corrections to the NRLMSIS-00 model decreased the reentry time prediction errors for satellites of the STARSHINE series by factors of 5.7, 3.0, and 1.6.

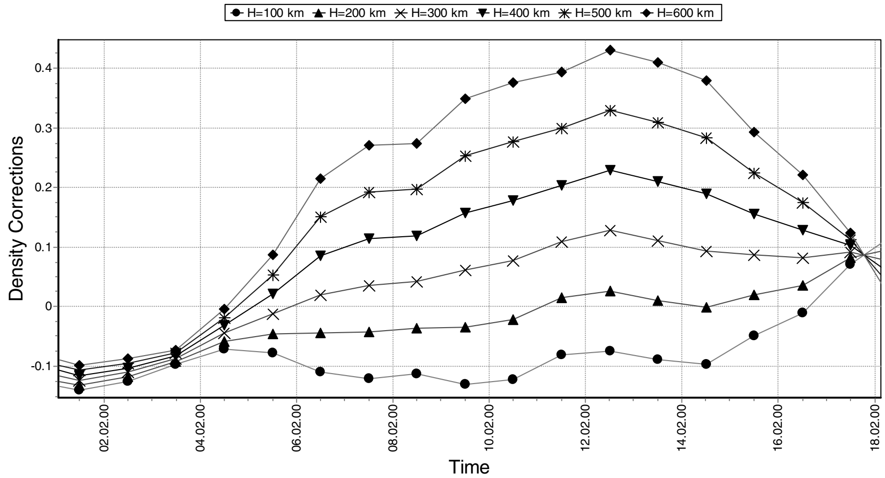

Fig. 4 Relative density corrections for February 2000.

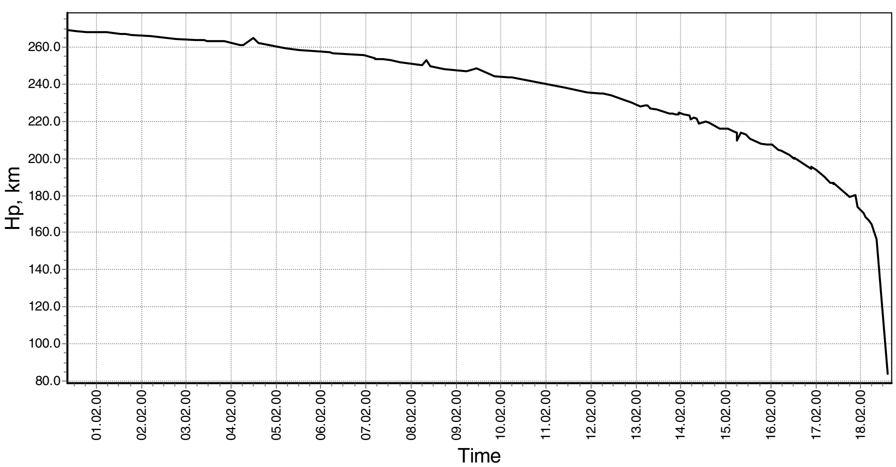

Fig. 5 Perigee altitude vs time for STARSHINE 1.

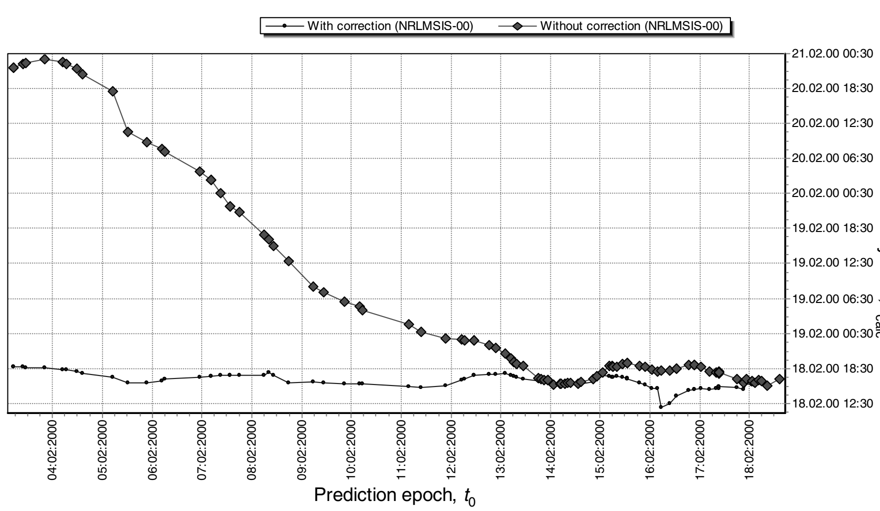

Fig. 6 Reentry time estimates for STARSHINE 1.

2) The rms values of the reentry time prediction errors were in the range of 4.9–8.1% before density corrections. These results are in good agreement with our estimates of the errors for the NRLMSIS-00 atmosphere model expressed by Eq. (2).

3) The residual level of the rms errors after density correction ranged from 1.4–3.1%; this is comparable to the errors in the density correction estimates for the NRLMSIS-00 model. This result is in good agreement with similar estimates for the spherical satellite 1980-37A [6].

IV. Arbitrary Shaped Space Objects

In addition to the satellites of the STARSHINE series, estimates of the effect of density corrections on the reentry time prediction

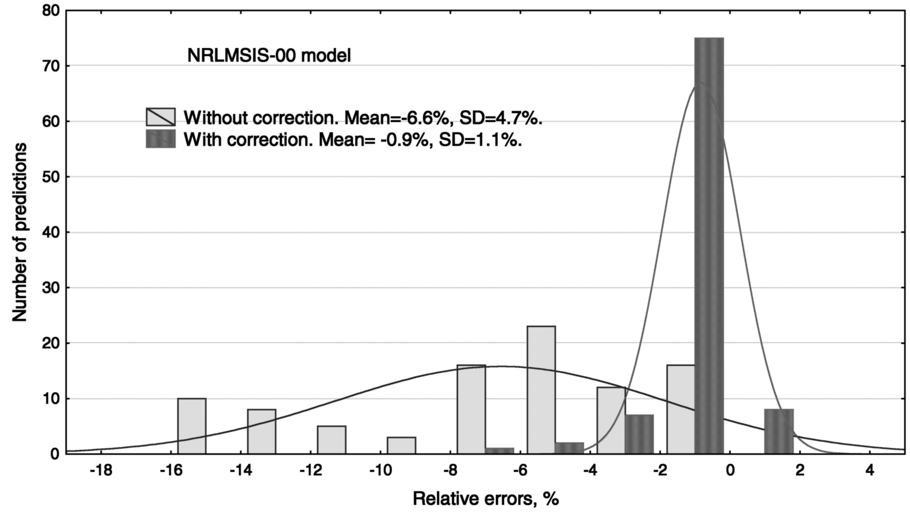

Fig. 7 Distribution of relative errors for STARSHINE 1.

accuracy were carried out for 95 LEO space objects having arbitrary shapes. For the majority of these objects, the prediction interval was limited to 10 days. The average number of predictions for SOs in this group was equal to 31. The average value of the rms errors in the reentry time for the case of predictions without density corrections was equal to 9.1%. The corresponding value of the rms after density correction equaled 6.9%, that is, a decrease of 32%.

Figure 8 shows the distribution of the ratios of the rms of errors calculated for each SO without density correction to those calculated with density corrections. This distribution characterizes, in a generalized form, the effect of the application of the density corrections. A value of the ratio greater than 1 on this plot signifies that accounting for the corrections resulted in an improvement in the accuracy of the reentry time prediction. A value of the ratio less than 1 implies that the density corrections resulted in a decrease in the accuracy. It is seen from this plot that for 72% of the SOs the reentry time prediction accuracy was improved. On the average, the density corrections resulted in increasing the prediction accuracy by a factor of 1.66.

For 27% of the objects the accuracy worsened after the density corrections. Let us consider the objects of this subgroup in more detail. Figure 9 presents a more detailed histogram of the distribution of the ratios for an rms lower than 1.0. For 40% (10 of 25) of the SOs of this subgroup, the rms errors were virtually identical before and after the density correction. The primary source of the reentry time prediction error for some of these SOs is not the density error, but other factors. For example, the variations of the ballistic coefficient of an SO due to its rotation around the center of mass can be a major factor. The analysis of the effect of this factor and techniques for taking it into account are considered in Sec. V.

The analysis of the results for SOs having an rms ratio value less than 0.7 has shown that these objects had rms error values before the density corrections that did not exceed 3%. This means that the reentry prediction accuracy was already very high for these SOs. A few percent decrease in the prediction accuracy after density corrections resulted in an observable change in the ratios of the rms in these cases. One of the probable reasons for this can be the errors in the determination of the density corrections at low altitudes over the time intervals corresponding to the reentry time of these SOs. This hypothesis requires additional verification, which can be a subject of a future investigation.

In conclusion, we should note that the capability of estimating the influence of the density corrections on the accuracy of reentry time predictions should be considered potentially achievable. They were calculated under the condition that we know the corrections to the density over the prediction interval. Under real conditions these corrections are unknown. Therefore, the real effects obtained in a real-time mode will be less. In this connection, one direction for improving the estimates in this mode can be the development of algorithms for forecasting the corrections to density and the improvement of atmosphere models. Yurasov et al. [7] illustrate the application of the autoregressive integrated moving average (ARIMA) model to the problem of forecasting the time series for the density corrections.

V. Space Object Individual Features

One of the reasons that accounting for the density corrections may not increase the reentry time prediction accuracy is that the SO's aerodynamic characteristics are changing. Such features can be caused, for example, by the long term character of the SO attitude motion. Variations of the ballistic coefficient value in some SO cases can become the main reason for motion prediction errors in the upper atmosphere. Analysis has shown that detection of such SOs is possible by comparison of their ballistic coefficient variations, obtained without and with density corrections.

Figure 10 presents the ballistic coefficient variations scatter plot obtained without and with the density corrections for the STARSHINE 3 satellite, which was a sphere. The comparison of these data sets indicates that the application of density corrections to the model has eliminated the long-periodic variations in the ballistic coefficient estimates caused by the errors in the atmospheric density model. Before density corrections, the standard deviation (SD) of the ballistic coefficient values was 12.8%; after application of the density corrections, the SD decreased to 2.8%.

Figure 11 shows the plots for estimates of the ballistic coefficient variations obtained without and with the atmospheric density corrections for SO 26124, which decayed on 9 January 2003. For this

Table 2 Statistics for reentry prediction errors for the STARSHINE satellites

		RMS of relative errors of reentry prediction, %		
SO no.	No. of predictions	Without density correction	With density correction	Ratio of RMS
25769	91	8.1	1.4	5.7
26929	65	6.5	2.1	3.0
26996	68	4.9	3.1	1.6

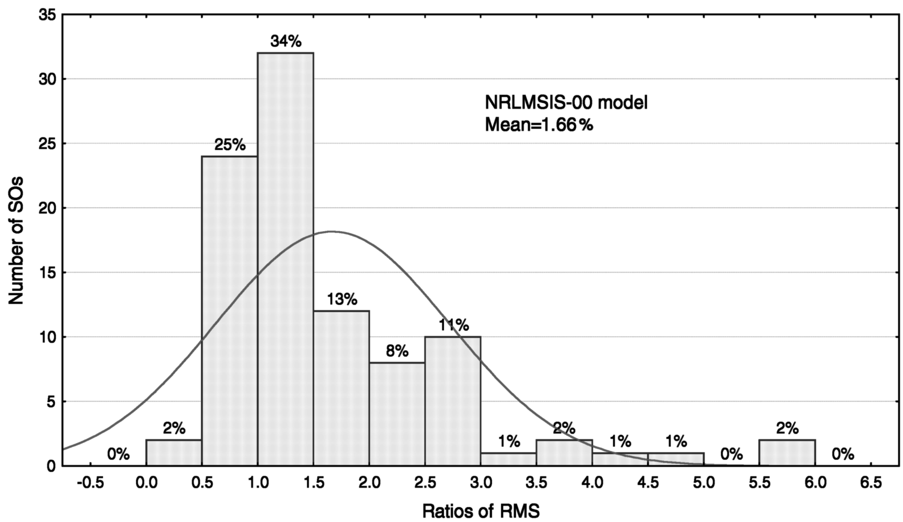

Fig. 8 RMS ratios distribution for 95 SOs.

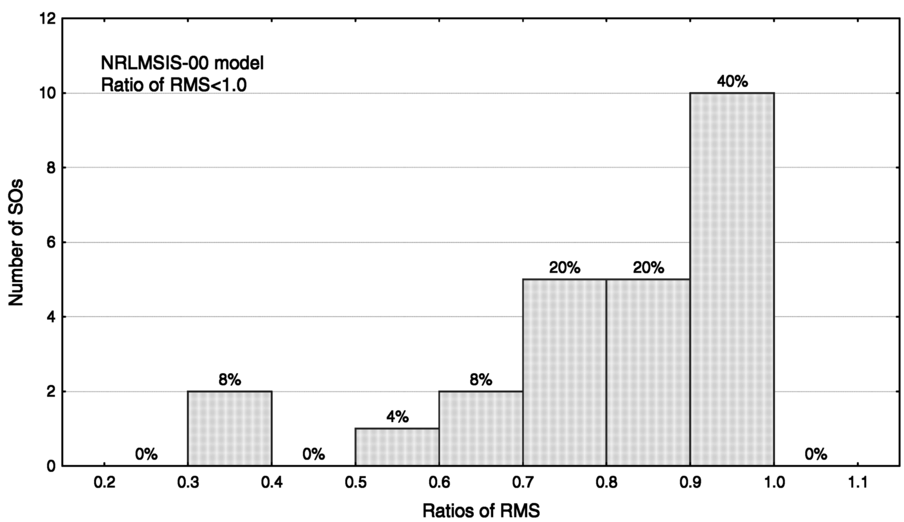

Fig. 9 RMS ratios distribution for subgroup of 25 SOs.

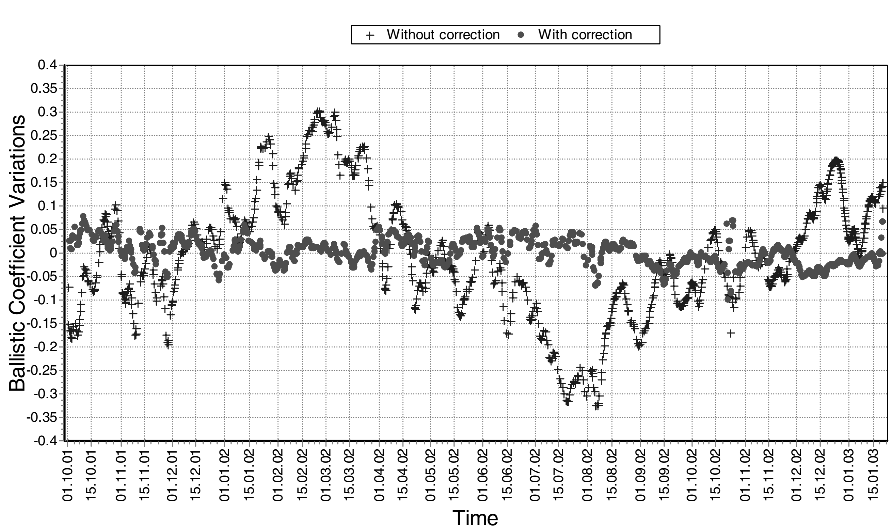

Fig. 10 Ballistic coefficient variations obtained for STARSHINE 3 without and with density corrections.

SO, the maximum estimates for the ballistic coefficient differ by a factor of 5 from the minimum ones, both before and after density corrections. Note that the ballistic coefficient variation has a dominant period of about 5.6 months.

Analysis has shown that all the decayed SOs of this kind are lightweight space debris and that they have large mean ballistic coefficient values (see Table 3) over the fit span. The common regularity in the ballistic coefficient variations of these SOs was their

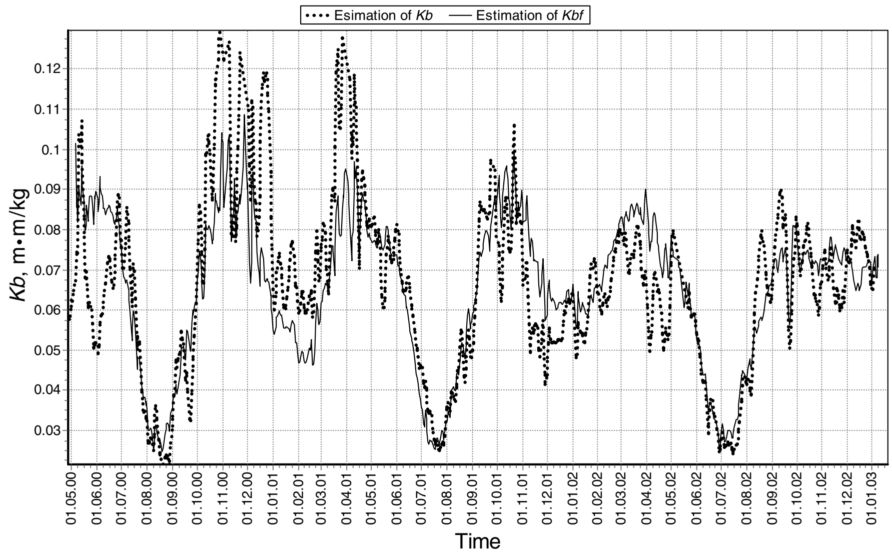

Fig. 11 Ballistic coefficient variations obtained for SO 26124 without (*Kb*) and with (*Kbf*) density corrections.

Table 3 SOs with large ballistic coefficient variations

SO no.	Decay date	Mean Kb, m^2/kg	Number of realizations	SD of Kb variations, % Before corrections	After corrections
17131	08 Dec. 2002	0.0638	424	24.5	32.6
24124	01 April .2003	0.1739	1955	30.4	26.3
24977	11 Oct. 2002	0.7813	1539	33.4	32.5
26124	09 Jan. 2003	0.0656	910	32.6	28.0
26428	20 May 2002	0.3214	835	40.3	28.9
27081	05 Sept. 2002	0.0624	300	37.5	37.9
27113	23 May 2003	0.0737	567	36.5	34.4
27145	05 Sept. 2003	0.0833	893	28.6	29.6

prominent periodicity with high amplitude, whose value was commensurate with the mean value of Kb. Along with the basic harmonic, whose period was different for various satellites and ranged from 8 days to 6.5 months, other periodic components were also present. The aerodynamic characteristics of individual SOs can be revealed by analysis of the histories of the ballistic coefficients obtained with density corrections. Further, these regularities can be used for forecasting the SO's ballistic coefficient variations and for the SO's reentry time prediction. The results obtained in this area show that prospective methods for the solution of this task can be an autoregression and ARIMA time series analysis and forecasting methods [6–9].

VI. Conclusions

Increasing the accuracy of satellite reentry time prediction is a complicated scientific and technological problem. To improve the accuracy, it is necessary to use a comprehensive approach to account for all of the significant factors.

The error in the upper atmosphere density model used in the satellite motion models can have a significant influence on the reentry time of SOs. One of the methods for decreasing the influence of this factor is determining current time corrections of the atmospheric density, and including these corrections in the prediction of the space object's reentry time. Using density corrections, the error in the reentry time prediction for spherical objects was significantly reduced. The ratio of the rms error of the reentry time prediction without density correction to the error with density correction ranged from 1.6 to 5.7. For nonspherical objects, the improvement in the reentry time prediction was not as good due to the time varying nature of the ballistic coefficient. The average value of the ratio for this group of objects was 1.32, an improvement of 32%. Thus, using the density corrections can provide an improvement in the reentry time prediction.

Analyzing the changes in the ballistic coefficient estimates, obtained with density correction, and finding the individual features of the evolution of the SO's aerodynamic characteristics also represent a direction for increasing the accuracy of SO reentry time prediction.

Potential directions of future work are 1) forecasting the atmospheric density corrections and their inclusion in the SO's reentry time determination, and 2) forecasting the SO's ballistic coefficient variations on the basis of revealed individual time regularities and their inclusion in the SO's reentry time determination.

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